The deep learning principles behind the introductory ramp in *Wuzzit Trouble:*
A study by Dr Anne Watson, University of Oxford

Introduced by Dr Keith Devlin, Chief Scientist, BrainQuake

The dramatic learning outcomes from a mere two hours of self-guided play of *Wuzzit Trouble* by children as young as the Third Grade, demonstrated by the recent Stanford University study,[1] is almost certainly a result of the confluence of several factors that well developed learning games can bring to bear, some of which I summarized in my article discussing the Stanford study,[2] and some others of which were discussed in the Stanford research report itself.

One evident factor is the design of the initial puzzle ramp. In order to ensure that early learners can progress to the sophisticated mathematical level required by the later stages of the game, and moreover do so with sufficient rapidity that the game retains their engagement, the initial ramp has to be designed with great care. The knowledge and skill required to do this greatly exceeds my own capacity, so when I was designing the *Wuzzit Trouble* puzzle sequence, I turned to a world famous early mathematics learning specialist to do the job for me. The first thirty-five puzzles are the result of her and her team's deliberations.

To a layperson, that sequence of puzzles may appear somewhat random — a series of elementary arithmetic problems that a teacher scribbled down in a few minutes in order to keep her class occupied. And to be sure, to anyone who has already mastered basic arithmetic, they offer little at all; indeed, parents or older children may report that they find the early stages of the *Wuzzit Trouble* game tedious. But from a learning perspective, they are far from that. We are unable to forget what we long ago mastered, and look at that puzzle ramp through the eyes of a first-time learner.

Whatever older children and adults think of the first thirty puzzles, with third graders demonstrating major learning gains in mathematical reasoning after just two hours of self-guided play, there is clearly something significant going on in that ramp. To determine what it is, I asked Professor Anne Watson, a world famous mathematics education scholar from the University of Oxford in the UK, to analyze the *Wuzzit Trouble* introductory ramp for me. Her report is reproduced below.[3]

Note that Professor Watson focuses on the introductory ramp — though at the end of her report she does make some comments about the later puzzles. The more advanced puzzles involve higher mathematical reasoning, a domain I myself am very familiar with, so I had no need to ask for outside analysis. I did however find it interesting to read Prof Watson's remarks on how the introductory ramp prepares the groundwork for the later, more advanced forms of mathematical reasoning I am familiar with. More kudos to the ramp designers.

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[1] Holly Pope, Jo Boaler, & Charmaine Mangram, *Wuzzit Trouble: The Influence of a Digital Math Game on Student Number Sense*, Stanford University, February 2015.

[2] Keith Devlin, Stanford study shows Wuzzit Trouble leads to significant math learning gains Major new research finding in game-based learning, April 2015, BrainQuake website.

[3] Prof Watson is a member of BrainQuake's Scientific Advisory Board. Following standard practice, Advisory Board members are unpaid, but given a nominal, very modest number of shares in the company as a token of appreciation. Their role is to assist the company by providing impartial expert advice. We chose our board to consist of world famous learning scientists who would not risk damaging their valuable reputations by being associated with anything less than world class science.

Pedagogy Analysis of the Introductory Learning Ramp in *Wuzzit Trouble*

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Nearly every national curriculum (the equivalent of common core in countries with national education policies) contains aims and statements about problem-solving, reasoning, and the connected nature of mathematics as well as statements about the importance of collaboration, persistence, disposition, and so on. Of course any more detailed descriptions of these that can have levels or grades attached to them are contentious. The best that most curricula have managed to come up with are statements about increased complexity, such as moving from one step problems that require application of unknown procedure through to multistep problems where the procedure is not obvious and may even have to be devised by the student.

Some interpretations of increased difficulty also relate to the numbers involved, but often this is reduced to the assumption that bigger numbers are harder than smaller numbers, decimal numbers are harder than whole numbers, and fractions are harder than . . . more or less everything.

Other interpretations of increased difficulty relate to the sophistication of the mathematical methods that might be used at each stage, and how straightforward or obscure it might be to recognise the variables and/or relationships to which those methods have to be applied. This latter kind of progression in problem solving means that you cannot be said to be an accomplished problem solver if you can only use low-level methods to do so. On the flipside of this is the fact that most of us, when we set out to solve problems in mathematics, try to use the simplest methods available, and even a good algebraist can end up using trial and reflection if that seems to be the easiest method available.

Bearing this in mind, I have analyzed the initial puzzle ramp in *Wuzzit Trouble* (the first 35 puzzles) to understand why it is so effective as a self-guided learning tool for children. This work will have been done at the design stage, of course. What I have been doing is reverse engineering the ramping.

It is important to describe the game in terms of how it builds on players' knowledge and experience, and how it provides the grounding for future experience of mathematics both within the game and outside it. More than that, there are specific places later on in the game (beyond the introductory ramp) where the player has to shift to using more sophisticated methods—it gets very tedious trying to play the Future Tech Lab using trial-and-error! [KEITH DEVLIN comment: With some later puzzles having a solution space with over 2 trillion paths, trial-and-error becomes impossible.]

By my analysis, the features of *Wuzzit Trouble* that relate most closely to aspects of any curriculum are, in progressive order, as follows:

1. Using 5 as the counting unit, so that fingers can be used and multiples of five can become familiar, adding and subtracting with small multiples of five in an efficient way to get a target answer. In addition to this, the counting in five has to be related not to objects, and not to a number line, but to a circular scale which is marked in fives — usefully for metric measure!

2. Players become more familiar with skip counting in fives, and have to tally their skip counts so that they can use multiples. This discourages "counting on in fives" and encourages "recognising multiples of five". This is an important step in understanding multiplication.

3. Hitting target numbers by working out the necessary multiples provides the groundwork for understanding multiplicative equations later on. Doing this in the context of 5 means that the players are working with the familiar number.

4. They then move to using combinations of numbers starting with a less familiar pair 4 and 7. This encourages the rejection of naive strategies of counting in fives, and assumes that players will have realised the importance of multiplication and will have to use multiplication facts as a toolbox, may be doing some side work to prepare themselves. The alternative is to count around the dial in 4s and 7s and keep a tally of how many have been used, but this will become tedious and the need for the toolbox will become obvious. This is a very different model of the use of multiplication facts than, say, a rectangular array of objects in which skip counting is an adequate strategy. *Wuzzit Trouble* discourages skip counting and encourages fluent use of facts.

5. Later on it becomes even more necessary to use multiplicative facts rather than relying on repeated addition.

6. Note that when they are using gears in pairs, some of the pairs are numbers that are closely related as one is a factor of the other; other pairs have gcd = 1. If players are encouraged to make up their own puzzles to play off-screen, they may find that they can make impossible puzzles using some of these pairs. This provides the ground work for later understanding of linear relationships, and also early number theory. While imagining that all players are going to study number theory in their free time is a bit fanciful, it is less fanciful to imagine that if the two cogs are both multiples of 4, then the only numbers that can be made are multiples of 4, and the only way to get away from that is due to the fact that we are working in mod 65

7. Later puzzles assume an accumulation of learned relationships about and between numbers, and a developing fluency in the use of the scale. Players become adept at looking at the distance between scale marks, without having to count round. Players also become adept at using negative numbers, and combinations of positive and negative numbers, even when the markings in a negative direction are not what you would find on a number line. In other words the concept of negativity has to override the symbolic information and this is an important step in mathematics because it is dependent on structure and not on numbers.

NOTE: So far, therefore, players are likely to have rejected counting on in ones; rejected skip counting; become more fluent in multiplication facts; become fluent in interpreting "distance between" even when negative numbers are involved; may have some understanding of what can be achieved with pairs of numbers that do/do not have factors in common; are likely to have understood that structure and relationships on the dial are sometimes more important than individual numbers.

NOTE: All this is in addition to the background increase in problem-solving skills and number fluency, as they move from familiar to less familiar numbers, and from simple positive movements to multistep positive and negative movements.

7. These developments continue and any residual reliance on skip counting has to disappear before confident progress can be made with the more complex versions of the game. In the initial ramp, ground work is definitely being laid for understanding linear connections, and there is a move away from learned multiplication facts towards a need to construct new sets of multiplication facts.

8. NOTE TO TEACHERS: A strategic move that might be made is that these new sets of multiplication facts to be written down and players to experiment, particularly of the working together with others, about how to combine multiples of these facts with each other to make useful numbers. Earlier observations about whether or numbers can possibly be made or not will be useful here.

9. NOTE TO TEACHERS: In a school environment, at this stage teachers might consider asking players to construct helpful notations to keep track of their reasoning; these would be versions of the equation

$$km + ln = p.$$

There is also the possibility of constructing graphical representations of $km + ln = ?$ And moving the resulting graph until p is achieved and reading off integer values from a coordinate lattice. It is unlikely that players will do this on their own, but this is what I mean by laying the groundwork for understanding later mathematics.

10. Much can be made of the applied linear structure of some of the later puzzles in which it is fruitful to search for how target points are related by multiples of one of the gear values. This reduces the problem to, for example, $5k$ + starting number so that what has to be found is how to get to the starting point from which the multiples can be generated. This seems to me to be an unusual way of presenting a linear expression — by moving the "origin" to a position from which multiples of only one gear need to be used.

In the above summary, I have avoided mentioning various number theory theorems which seem to be lurking in the background of the game, and have also avoided using the name "Diophantus'" who hangs over all of this like a benign ghost. Instead I have tried to connect the various shifts in the game to the curriculum, and how experience of the game could be drawn on by teachers to make sense of later algebra.

My comments have been heavily influenced by recent tightly focused one-to-one work with students who are not flourishing in mathematics, and my discovery that a lot of this — among those who are within the normal cognitive compass — is to do with their ability to count on quickly and accurately in all arithmetical contexts and therefore avoid understanding the additive and multiplicative structures. This falls apart not only when doing written arithmetic — because who cares about that? — but when understanding how multiplication is used and in being able to use relationships between numbers, e.g. difference, scaling etc. They therefore have nothing to build on for algebra either. *Wuzzit Trouble* attacks these tendencies to become over fluent in overly simple approaches.

Looking at the later puzzles, beyond the introductory ramp, one thing the multi-cog puzzles do is draw attention to the multiplicative relationship between numbers. This sense of relationship is essential for understanding equivalence in ratios and equivalence of linear functional relationships.

The other thing the multi-cog puzzles introduce is this notion of controlling variables, so that at any stage one of them becomes the constant and the other the variable for a linear relation. This provides the basis of extensive experience in constructing input-output generalizations, so therefore provides some groundwork for understanding algebraic functions and for encouraging players to devise what are called correspondence approaches to constructing these. I am thinking of those typical function machine diagrams in textbooks where children are expected to work out what the operations are and in what order they happen in order to get the expected output. *Wuzzit Trouble* provides a context for having to pay attention to order of operations, particularly when players are doing written side work to devise efficient strategies.

I am also of the opinion that and other aspects of mathematics for which this game provides the groundwork are linear analysis, and the relationship between parameters and variables in systems of linear equations, but this would require further analysis.